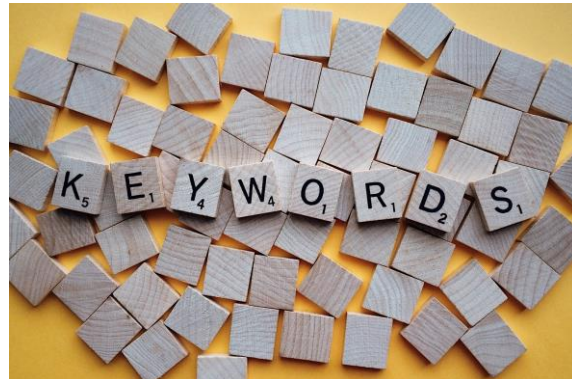


# Mathematics education at Dutch primary schools: a journey from answer-getting to problem-solving

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## Key words:

Higher order thinking skills

Problem solving

Teacher skills

Primary teacher education



# What is the state of mathematics education in the Netherlands?



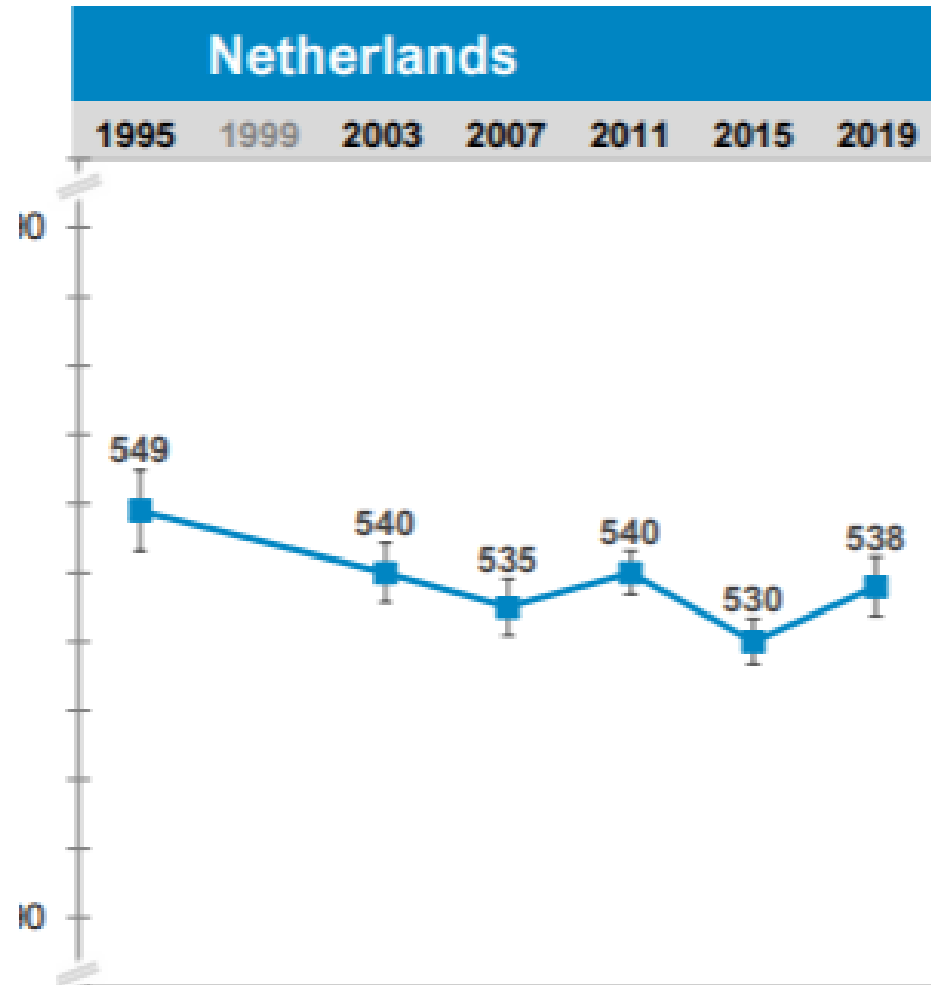
13 April 2022:  
Chief inspector Alida Oppers hands over to ministers of education: 'De staat van het onderwijs'

Major concerns about education in the Netherlands

How is mathematics education doing?



# Results of TIMSS, 2019



TIMSS (2019)  
Trends in international  
mathematics and science  
study

Students of grade 4  
(10 years)

(Meelissen et al., 2020)

Landen	Gemiddelde score rekenen (s.e)	Schaalscore rekenen
Singapore	625 (3,9) ▲	
Hong Kong	602 (3,3) ▲	
Zuid-Korea	600 (2,2) ▲	
Chinees Taipei	599 (1,9) ▲	
Japan	593 (1,8) ▲	
Russische Federatie	567 (3,3) ▲	
Noord-Ierland	566 (2,7) ▲	
Engeland	556 (3,0) ▲	
Ierland	548 (2,5) ▲	
Letland	546 (2,6) ▲	
Noorwegen (grade 5)	543 (2,2) ▲	
Litouwen	542 (2,8) ▲	
Oostenrijk	539 (2,0) ▲	
<b>NEDERLAND</b>	<b>538 (2,2) ▲</b>	
Verenigde Staten	535 (2,5) ▲	
Tsjechië	533 (2,5) ▲	
België (Vlaanderen)	532 (1,9) ▲	
Cyprus	532 (2,9) ▲	
Finland	532 (2,3) ▲	
Portugal	525 (2,6) ▲	
Denemarken	525 (1,9) ▲	
Hongarije	523 (2,6) ▲	
Turkije (grade 5)	523 (4,4) ▲	
Zweden	521 (2,8) ▲	
Duitsland	521 (2,3) ▲	
Polen	520 (2,7) ▲	
Australië	516 (2,8) ▲	
Azerbeidzjan	515 (2,7) ▲	
Bulgarije	515 (4,3) ▲	
Italië	515 (2,4) ▲	
Kazachstan	512 (2,5) ▲	
Canada	512 (1,9) ▲	
Slowakije	510 (3,5) ▲	
Kroatië	509 (2,2) ▲	
Malta	509 (1,4) ▲	
Servië	508 (3,2) ▲	
Spanje	502 (2,1)	
<b>TIMSS-gemiddelde</b>	<b>500</b>	
Armenië	498 (2,5)	
Albanië	494 (3,4)	
Nieuw-Zeeland	487 (2,6) ▼	
Frankrijk	485 (3,0) ▼	

Results of  
TIMSS, 2019

# Results of TIMSS

## Trends in referentieniveaus rekenen

	Percentage of students					
Referentiepunten	1995	2003	2007	2011	2015	2019
Advanced level	12	5	7	5	4	7
High level	50	44	42	44	37	44
Middle level	87	89	84	88	83	84
Basic level	99	99	98	99	99	98

# Peil.Rekenen-Wiskunde (research of inspection of education)

Mathematical level at the end of primary school



## **Peil.Rekenen-wiskunde**

Research Inspection of education in 2018-2019  
Mathematical level at the end of primary school

(Inspectie van het onderwijs, 2021)

# Results of Peil.Rekenen-Wiskunde (research of inspection of education)

	Ambition Meijerink	Inspection research	Test end prim.ed.
1F	85%		
1S	65%		

Dutch students at the end of primary education (grade 6):

- reach the 1F-level,
- but the ambition of the 1S-level is not achieved.



## **Peil.Rekenen-wiskunde**

Research Inspection of education in 2018-2019  
Mathematical level at the end of primary school

(Inspectie van het onderwijs, 2021)

# Explanation for these results?

In Dutch primary mathematics education, teachers pay ...

much attention to practising,  
knowing what and knowing how...



less attention to developing  
mathematical higher order  
thinking skills, knowing why,  
(of all students)

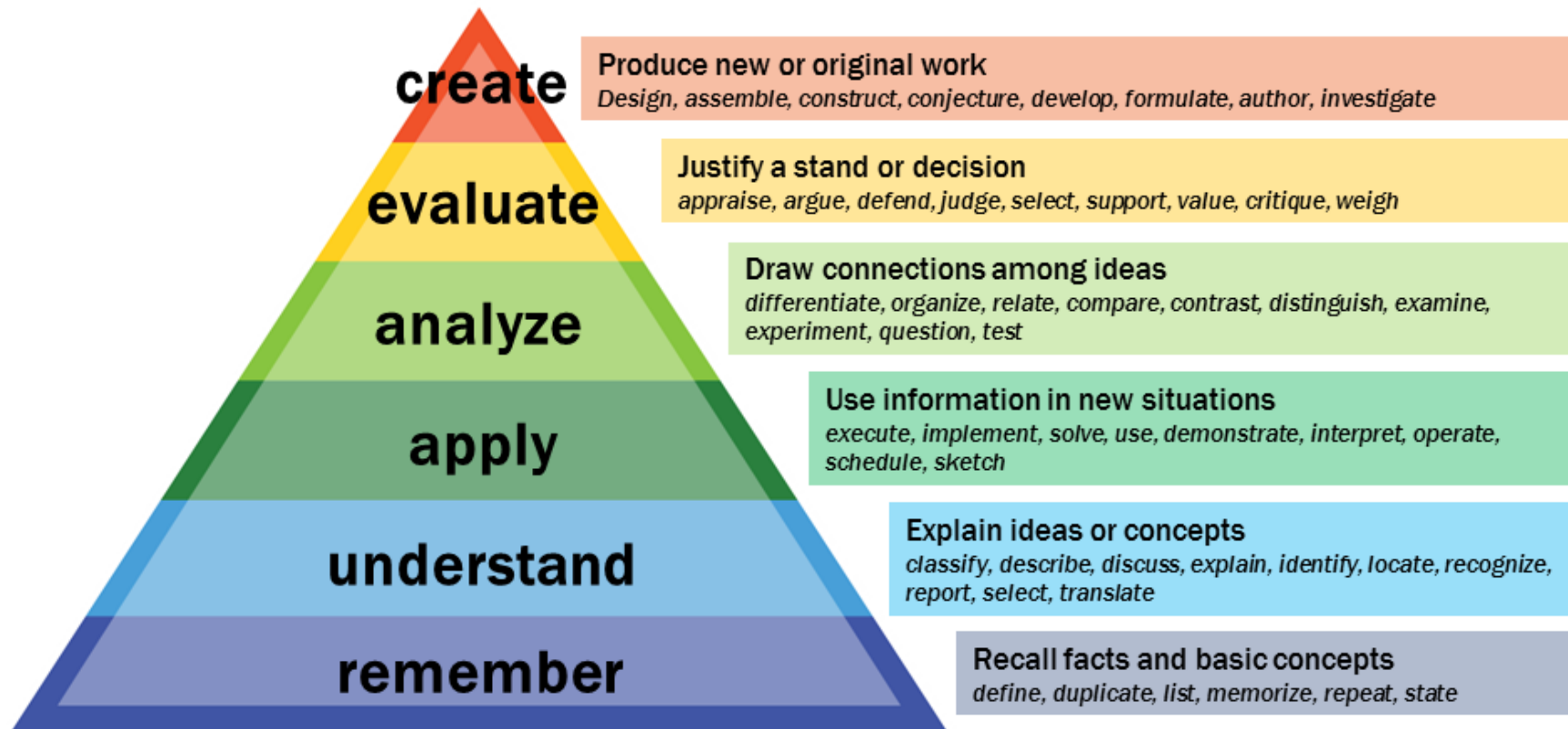




# Mathematical higher order thinking skills

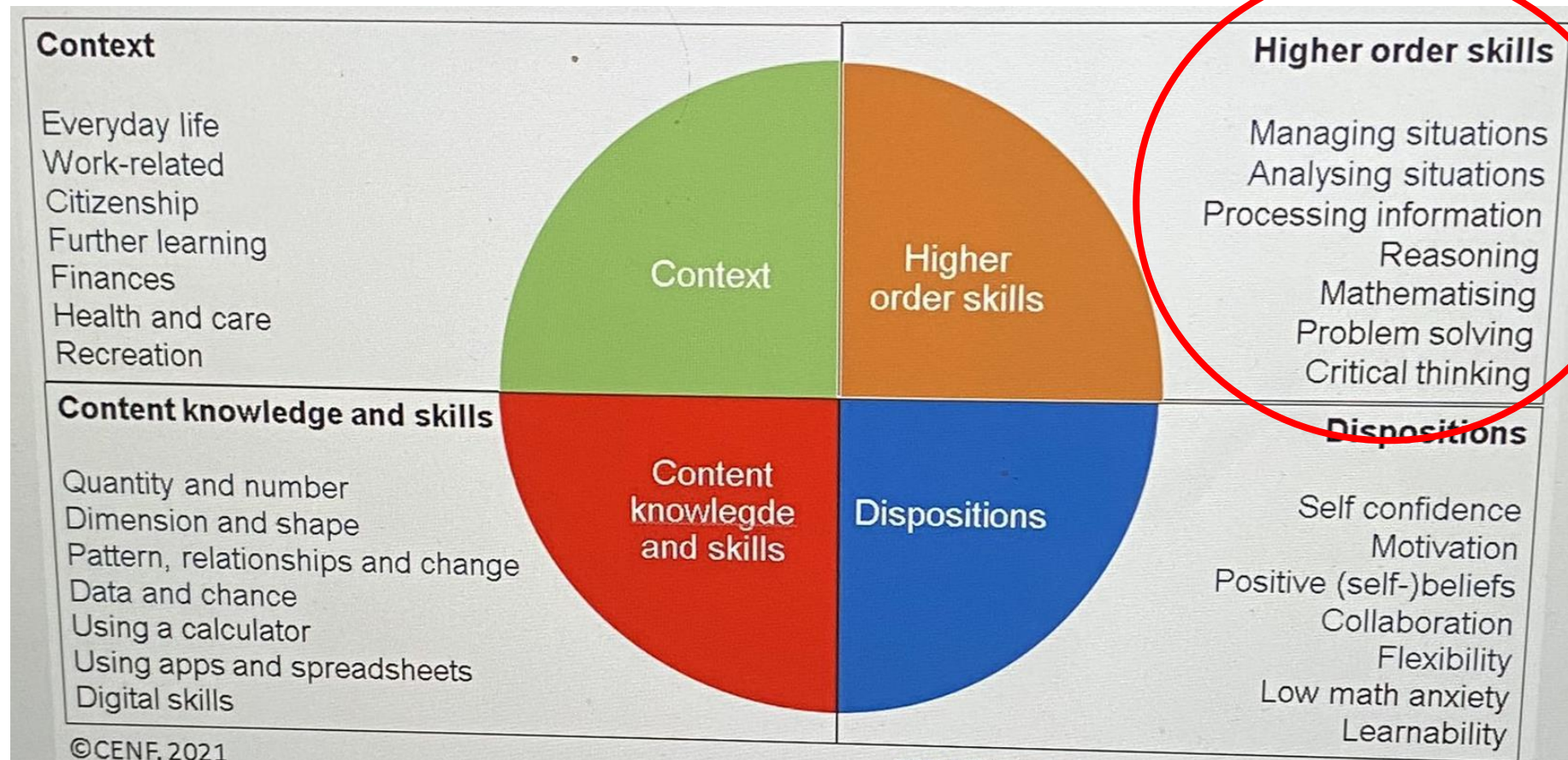


## Bloom's Taxonomy

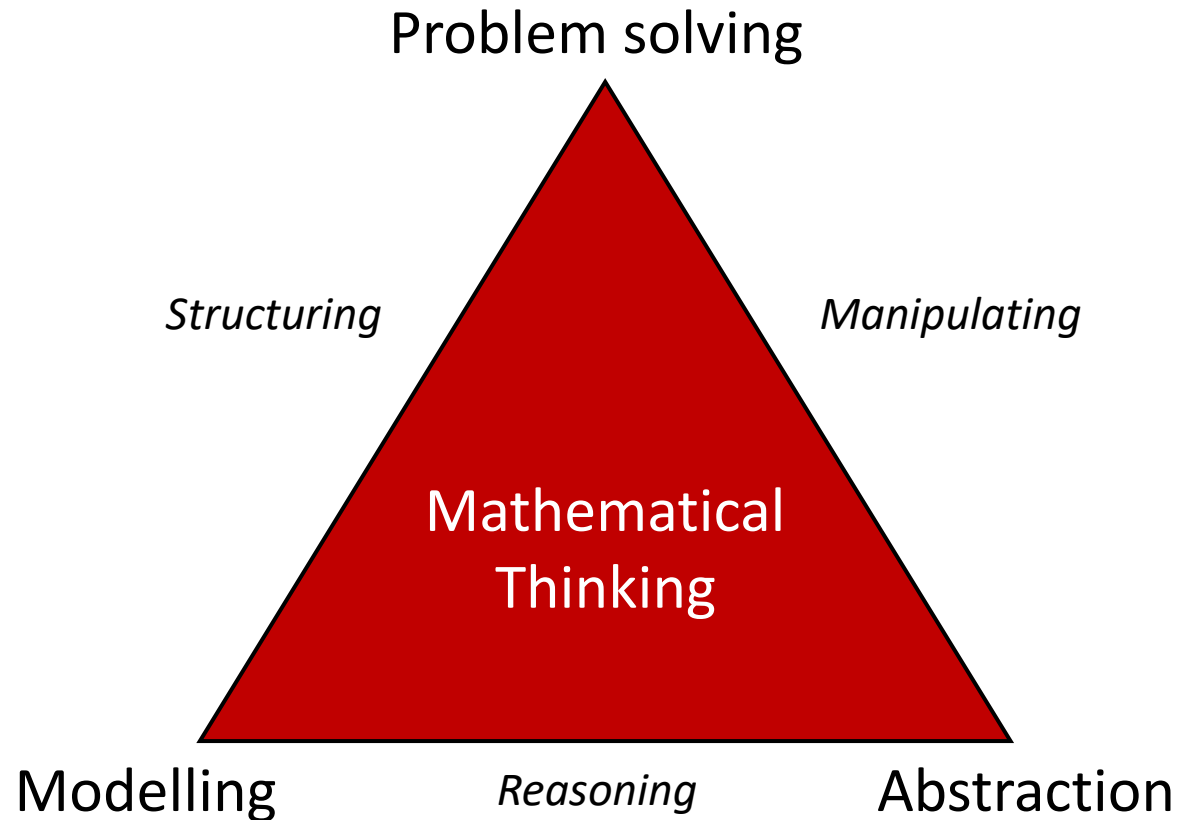


(Anderson, et al., 2001)

# Mathematical higher order thinking skills

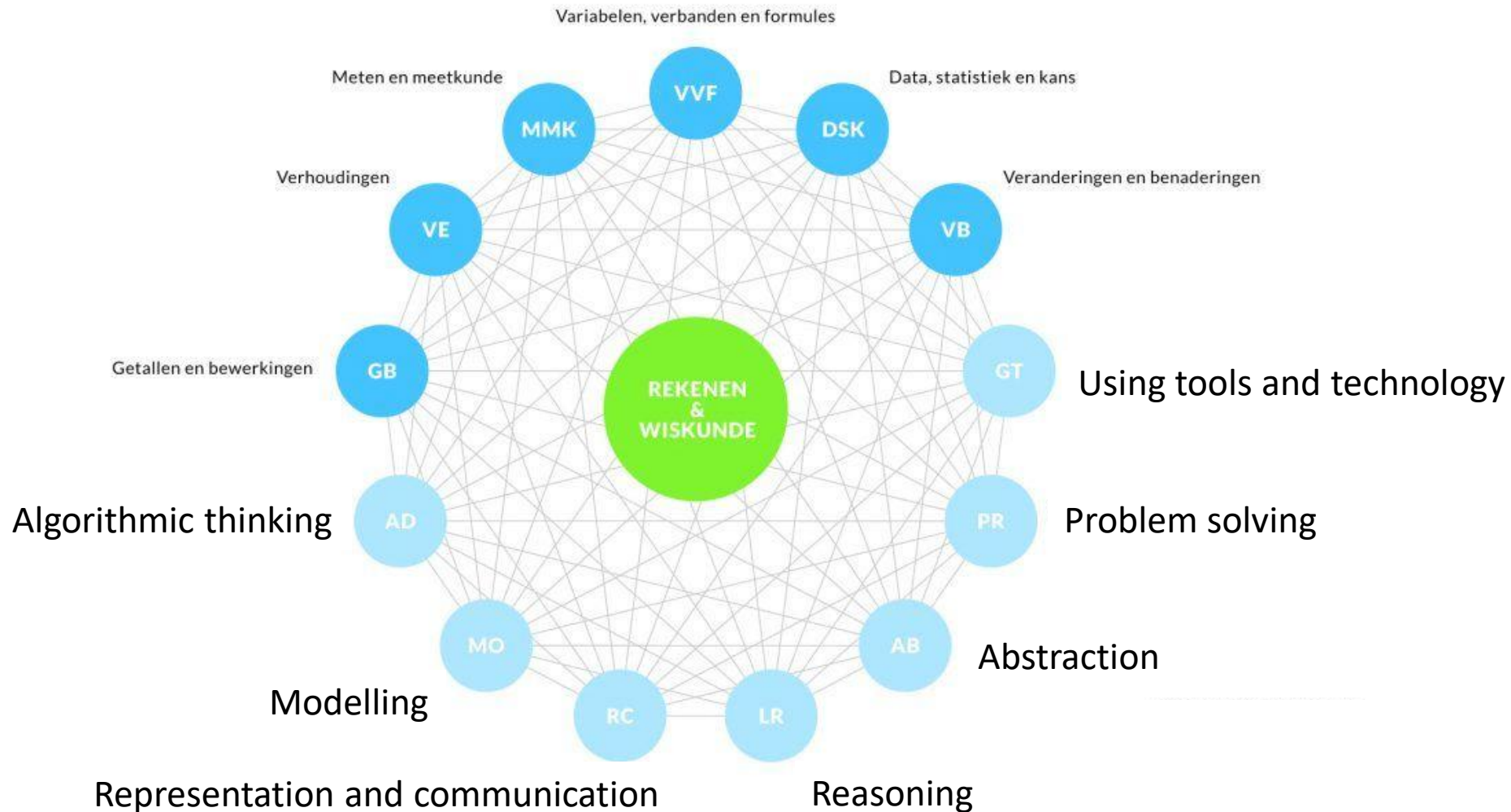


# Mathematical higher order thinking skills



(Drijvers, 2015)

# Mathematical higher order thinking skills



(Curriculum.nu, 2019)

# Why do mathematical HOTS get too little attention in Dutch primary education?



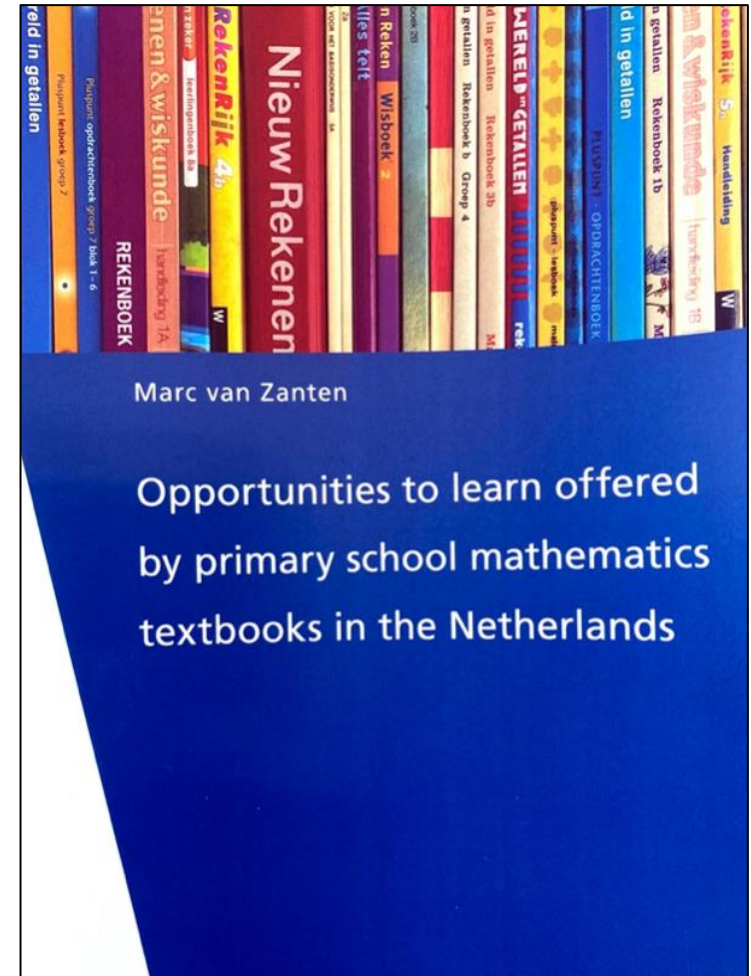
- Teaching basic skills and basic knowledge takes too much time.
- Teachers think only gifted students need to develop mathematical HOTS.
- Teachers think mathematical HOTS do not belong in primary education.
- It is hard to test mathematical HOTS in a pen and paper test.
- It is hard to develop HOTS when you are working with a digital textbook.
- Developing mathematical HOTS hardly appears in the Dutch textbooks for primary education.
- Teaching HOTS is challenging for the teachers. They need specific knowledge, skills and self-confidence.

# Why do mathematical HOTS get too little attention in Dutch primary education?



Dutch mathematical textbooks in primary education offer less opportunities to learn higher order thinking skills.

(Van Zanten, 2020)

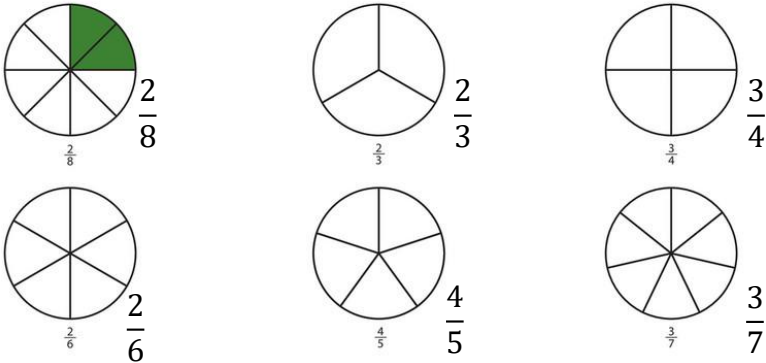


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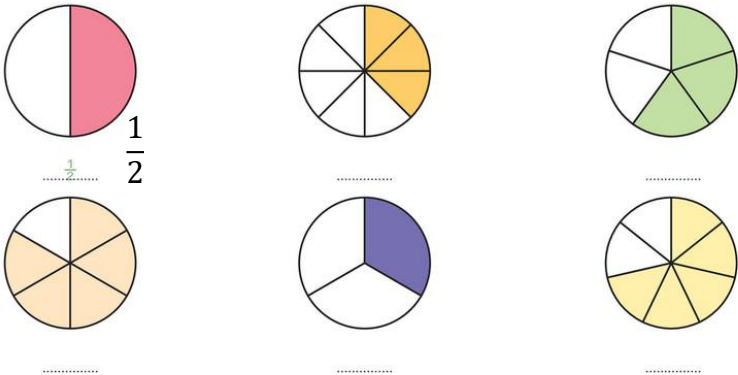


## Blok 5.4 17 Maatwerk

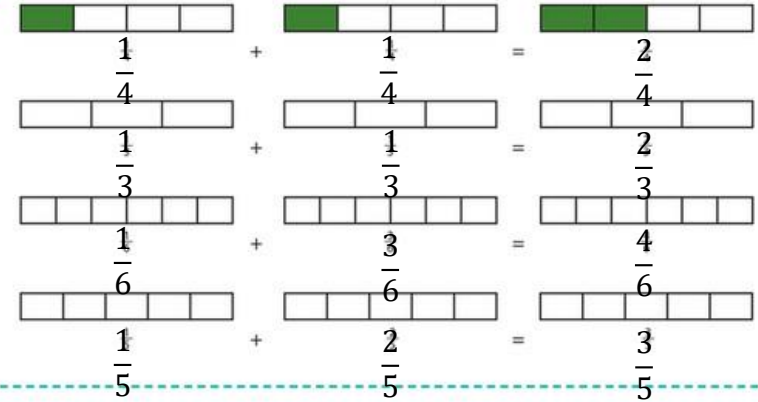
1 Kleur het aangegeven deel.



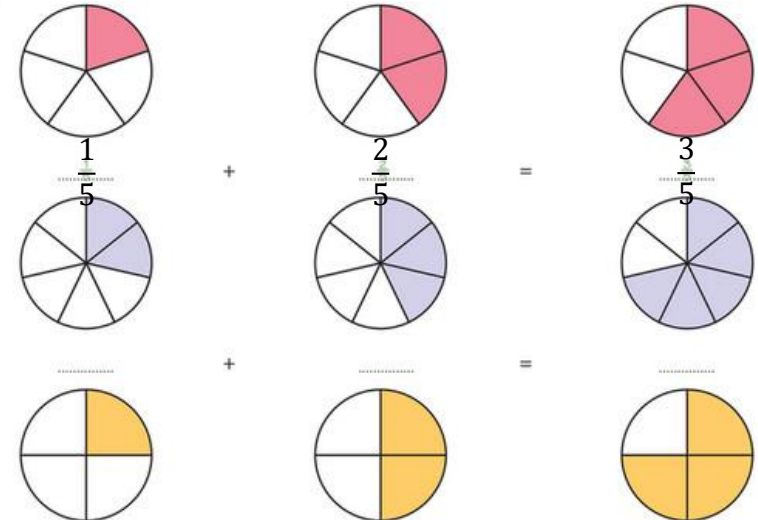
2 Welk deel is gekleurd?



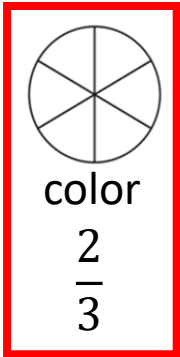
3 Kleur de som.



4 Schrijf de som op.



In Dutch mathematics textbooks much attention is paid to practise basic knowledge and skills

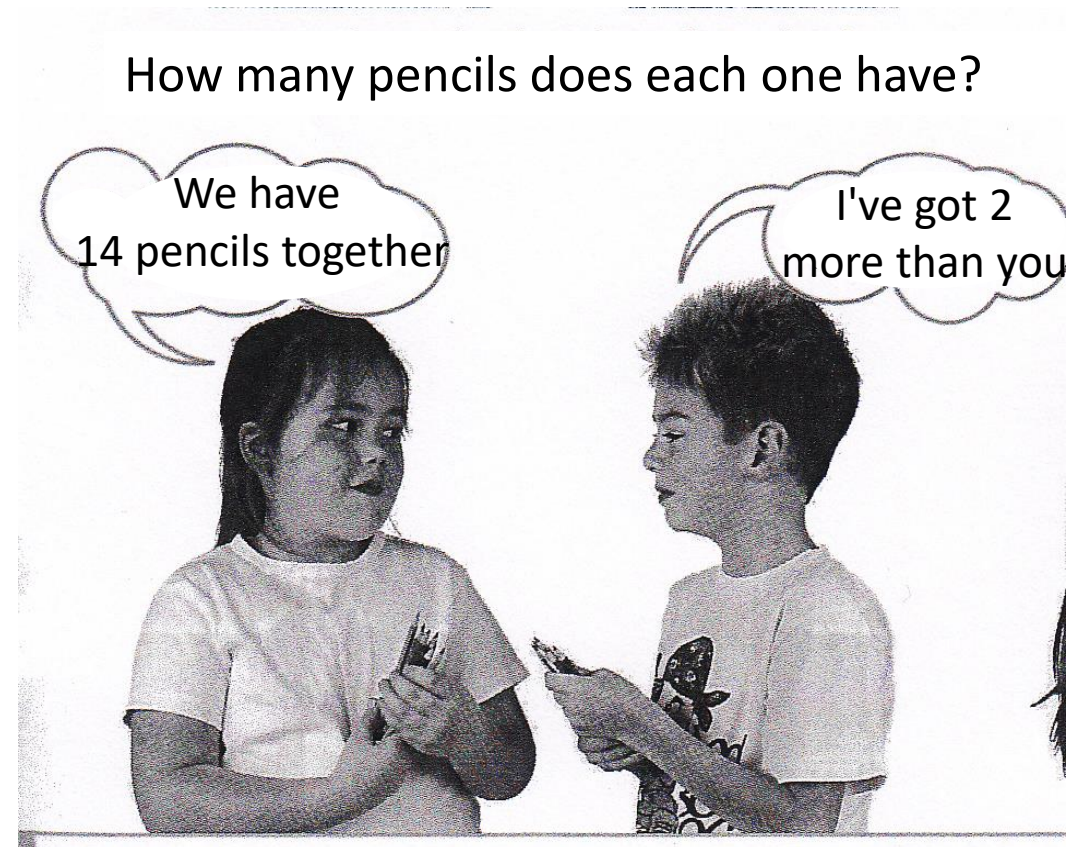


# Why do mathematical HOTS get too little attention in Dutch primary education?



Tweelingsommen.

Hoe verdeel je het eerlijk? Schrijf het in de poppetjes.



# Mathematical HOTS in primary education

Ways to enrich your mathematics textbook



The yearly big math day for primary education



2022

Building adventures

<https://www.onderwijsvanmorgen.nl/home/basisonderwijs/grote-rekendag/>

# Mathematical HOTS in primary education

Ways to enrich your mathematics textbook

Use math glasses to look at everyday life



'Real life rekenen' for grade 4 to 6

Publisher: Zwijsen



<https://www.nieuwsbegrip.nl/nieuwsrekenen>



<https://www.nieuwsindeklas.nl/>

# Mathematical HOTS in primary education

Ways to enrich your mathematics textbook



Marc Beaumont cycled around the world in 80 days.  
Could that be true?

<https://road.cc/content/review/252895-around-world-80-days-mark-beaumont>

# Mathematical HOTS in primary education

Ways to enrich your mathematics textbook

**Transform your students into critical fact checkers**

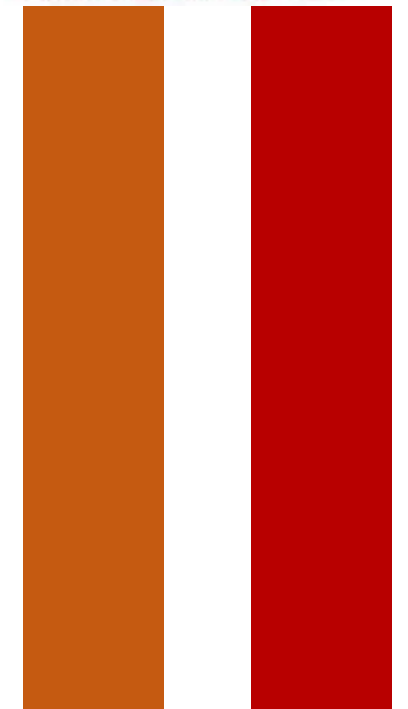
Adres				
№-E	Aantal	Artikel	Prijs	€
MAES	1x	Merrel 39		
RSN E		sierra blue.	85,-	
TLU:				
PASNR				
EXP. D				
BETAL				
DATUM:				
TRANSN				
TOTAAL				
U H				
RUILEN MET BDN IN ORIGINELE DOOS BINNEN 7 DAGEN				
B.T.W. _____				
Verk.	Datum	TOTAAL		
	28/7/2014	60,-		
33-318847		Wilt u bij reklame of ruiling deze koopbon meebrengen?		

The assistant of the shoe shop had to give me a 25% discount. Is her calculation correct?

At what price will her strategy produce a correct result?



**Discount on your energy bill**



**Discount on your energy bill**

# Mathematical HOTS in primary education

Ways to enrich your mathematics textbook



## The international Kangaroo competition



Competition about non-routine mathematics problems

Which picture has the most gray?



Math Kangaroo competition,  
2020, grade 2

# Why do mathematical HOTS get too little attention in Dutch primary education?



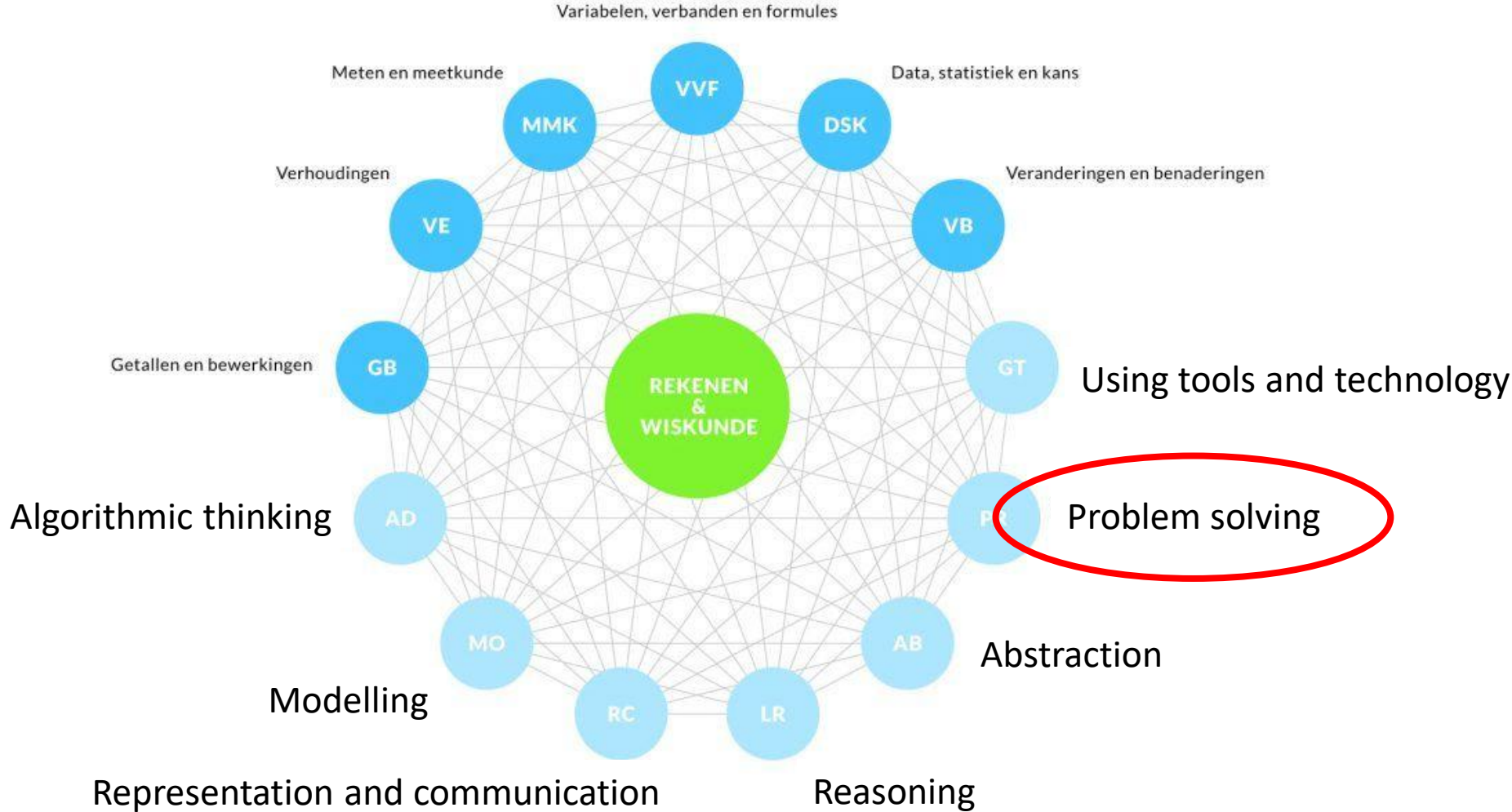
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- It is hard to develop HOTS when you are working with a digital textbook.
- **Developing mathematical HOTS hardly appears in the Dutch textbooks for primary education.**
- Teachers need specific knowledge, skills and self-confidence to teach mathematical HOTS.

# Why do mathematical HOTS get too little attention in Dutch primary education?



- Teaching basic skills and basic knowledge takes too much time.
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# Mathematical problem solving in primary education



(Curriculum.nu, 2019)



# Mathematical problem solving in primary education



## Problem 1: Airport Amsterdam - Schiphol

A ground stewardess notes the number of travelers lined up in front of her counter. She accidentally puts an extra zero after that number. This adds 198.000 to the actual number. Which number should she have written down?

22.000



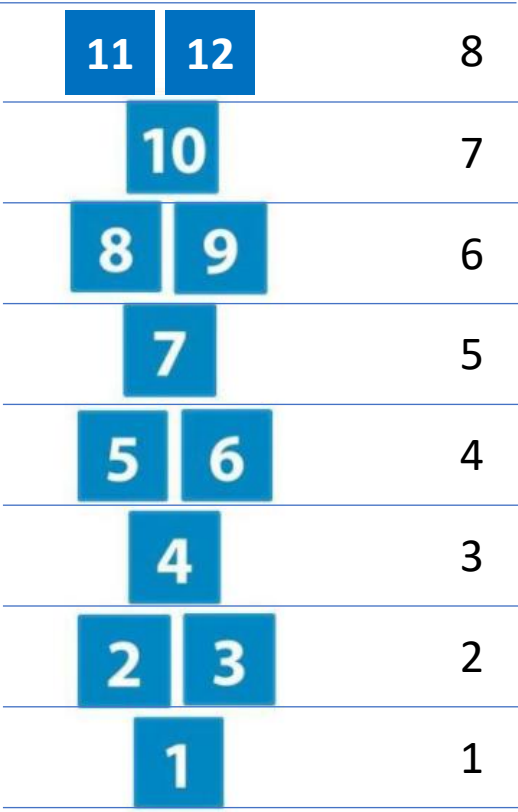
# Mathematical problem solving in primary education



## Problem 2: Hopscotch

This is a hopscotch  
It contains the numbers from 1 to 10 and its length is 7 tiles.

What is the length of this hopscotch if we add the numbers 11 and 12?



Length in tiles



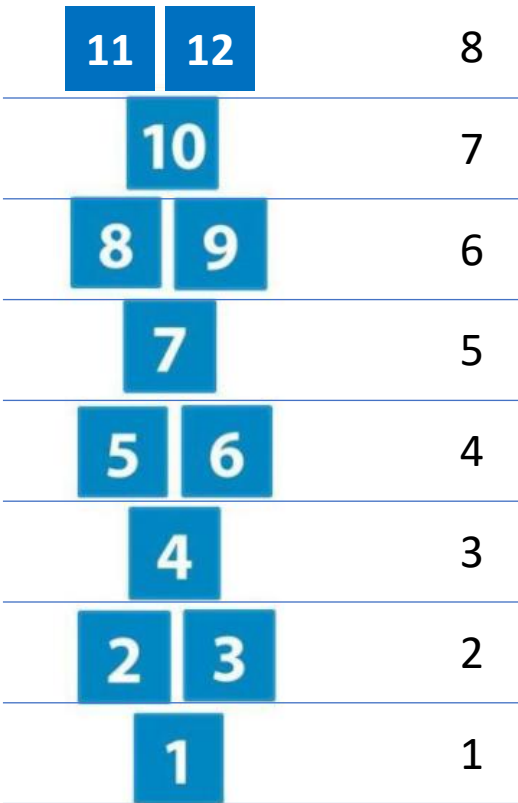
# Mathematical problem solving in primary education



## Problem 2: Hopscotch

I have a hopscotch that contains the numbers from 1 to 100.

What is the length in tiles of this hopscotch?

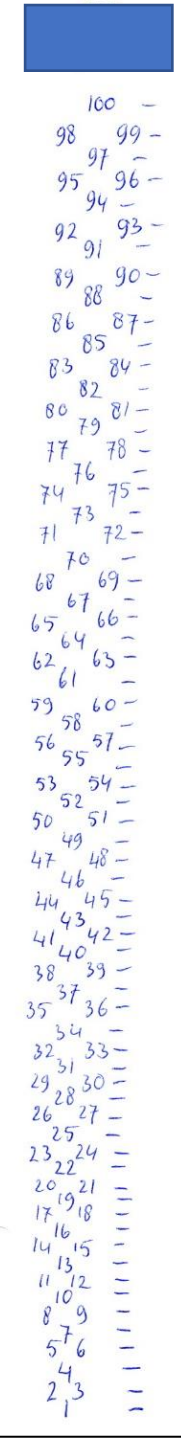


Length in tiles



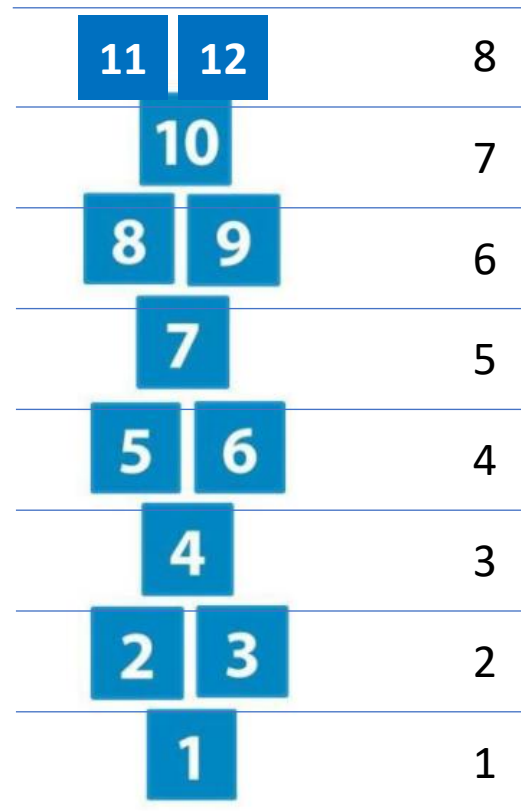
# Mathematical problem solving in primary

Education



Problem 2: Hopscotch

What is the length of a hopscotch with numbers from 1 to 100?



# Mathematical problem solving in primary

67

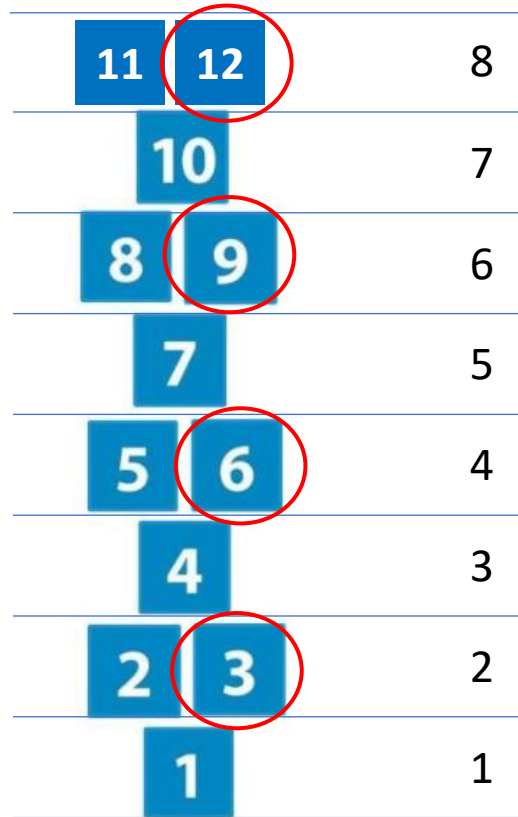
100 -  
98 99 -  
95 97 96 -  
94 -  
92 91 93 -  
89 90 -  
88 -  
86 85 87 -  
83 84 -  
82 -  
80 79 81 -  
77 78 -  
74 76 75 -  
71 73 72 -  
70 -  
68 67 69 -  
65 64 66 -  
62 63 -  
61 -  
59 60 -  
56 58 57 -  
55 -  
53 54 -  
50 52 51 -  
47 49 48 -  
44 46 45 -  
41 43 42 -  
38 40 39 -  
35 37 36 -  
34 -  
32 31 33 -  
29 30 -  
26 28 27 -  
25 -  
23 24 -  
20 22 21 -  
17 19 18 -  
14 16 15 -  
11 13 12 -  
8 10 9 -  
5 7 6 -  
4 -  
3

Education



## Problem 2: Hopscotch

What is the length of a hopscotch with numbers from 1 to 100?



Problem approach 2

Hopscotch with numbers to ...	Length in tiles...
3	2
6	4
9	6
12	8
15	10
18	12
99	...?

x 33

x 33

A hopscotch with numbers to 99 has a length of 66 tiles. Add one tile to find the length for a hopscotch that ends on 100.

# Mathematical problem solving in primary

Problem approach 1

67

100 -  
98 99 -  
95 97 96 -  
94 -  
92 91 93 -  
89 90 -  
88 -  
86 87 -  
85 -  
83 84 -  
82 -  
80 79 81 -  
77 78 -  
76 -  
74 75 -  
73 -  
71 72 -  
70 -  
68 69 -  
67 -  
65 66 -  
64 -  
62 63 -  
61 -  
59 60 -  
58 -  
56 57 -  
55 -  
53 54 -  
52 -  
50 51 -  
49 -  
47 48 -  
46 -  
44 45 -  
43 42 -  
41 40 -  
38 39 -  
37 -  
35 36 -  
34 -  
32 33 -  
29 30 -  
28 -  
26 27 -  
25 -  
23 24 -  
22 -  
20 21 -  
19 -  
17 18 -  
16 -  
14 15 -  
13 -  
11 12 -  
10 -  
8 9 -  
7 -  
5 6 -  
4 -  
2 3 -  
1

Education



Problem approach 3

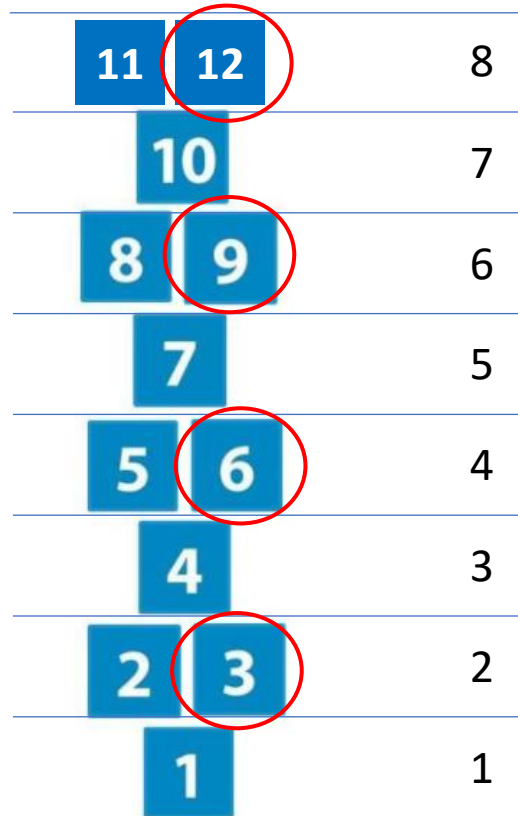
A hopscotch to 3 has a length of  $\frac{2}{3} \times 3 = 2$  tiles.

A hopscotch to 99 has a length of  $\frac{2}{3} \times 99 = 66$  tiles.

A hopscotch to 100 has a length of  $66 + 1 = 67$  tiles.

Problem 2: Hopscotch

What is the length of a hopscotch with numbers from 1 to 100?



Problem approach 2

Hopscotch with numbers to ...	Length in tiles...
3	2
6	4
9	6
12	8
15	10
18	12
99	...?

x 33

A hopscotch with numbers to 99 has a length of 66 tiles. Add one tile to find the length for a hopscotch that ends on 100.

# Mathematical problem solving in primary

Problem approach 1

## Problem 2: Hopscotch

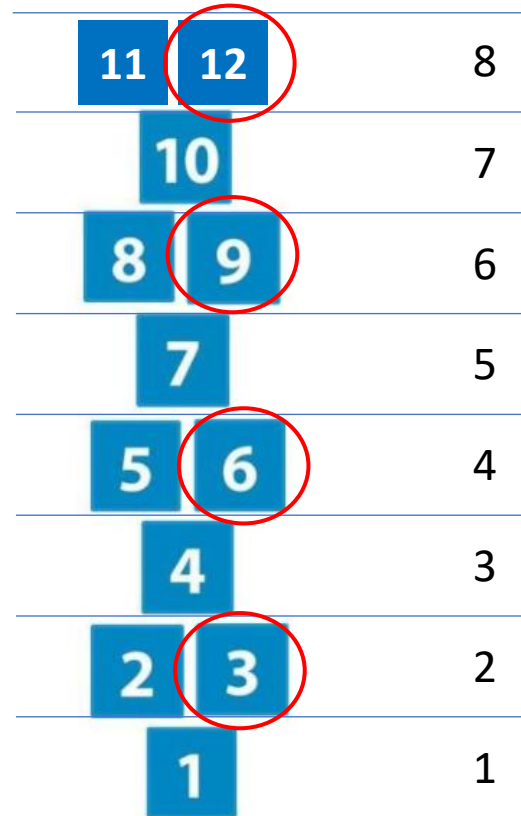
What is the length of a hopscotch with numbers from 1 to 100?

Problem approach 2

Hopscotch with numbers to ...	Length in tiles...
3	2
6	4
9	6
12	8
15	10
18	12
99	...?

x 33

x 33



67

100 -  
98 99 -  
95 97 96 -  
92 94 -  
91 93 -  
89 90 -  
88 -  
86 87 -  
85 -  
83 84 -  
82 -  
80 81 -  
77 79 78 -  
74 76 75 -  
71 73 72 -  
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65 67 66 -  
62 64 63 -  
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59 60 -  
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56 57 -  
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53 54 -  
52 -  
50 51 -  
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47 48 -  
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44 45 -  
41 43 42 -  
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38 39 -  
37 36 -  
35 -  
34 -  
32 31 33 -  
29 28 30 -  
26 27 -  
25 -  
23 24 -  
22 -  
20 19 21 -  
17 18 -  
16 -  
14 15 -  
13 -  
11 12 -  
10 9 -  
8 7 6 -  
5 4 3 -  
2 1

Education

Problem approach 3

A hopscotch to 3 has a length of  $\frac{2}{3} \times 3 = 2$  tiles.

A hopscotch to 99 has a length of  $\frac{2}{3} \times 99 = 66$  tiles.

A hopscotch to 100 has a length of  $66 + 1 = 67$  tiles.

Problem approach 4

The length to 100 is  $10 \times 7 = 70$  tiles

What is going wrong?



A hopscotch with numbers to 99 has a length of 66 tiles. Add one tile to find the length for a hopscotch that ends on 100.

# Mathematical problem solving in primary education



Problem Solving Strategies							
<b>Draw a picture</b> 	<b>Guess and check</b> 						
<b>Make a list</b> 	<b>Make a table</b> <table border="1"> <thead> <tr> <th>A</th> <th>B</th> <th>C</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> <td>3</td> </tr> </tbody> </table>	A	B	C	1	2	3
A	B	C					
1	2	3					
<b>Act it out</b> 	<b>Work backwards</b> start from the end						
<b>Write a number sentence</b> $10 + 4 = 14$	<b>Use objects</b> 						

11	12	8
10		7
8	9	6
7		5
5	6	4
4		3
2	3	2
1		1



In this table you see some different problem solving strategies. Which one(s) did you use to solve the hopscotch problem? What do you think about it now? Did you make a good choice? Why?



# Mathematical problem solving in primary education



Problem 1: Airport Amsterdam - Schiphol



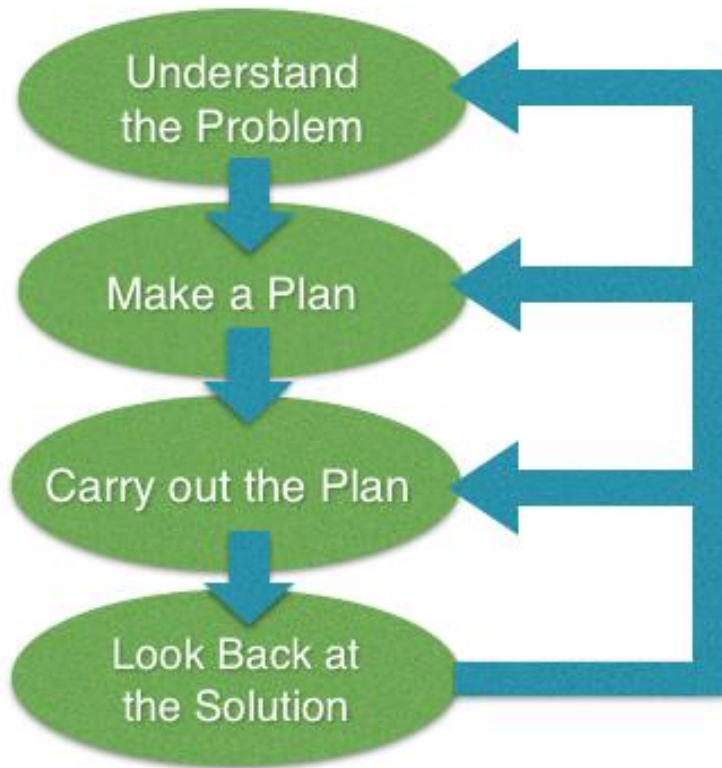
Problem 2: Hopscotch



In which of the two tasks did you feel more comfortable and stimulated to get started?

Which task was the most valuable? And why do you think so?

# Mathematical problem solving in primary education



Polya's Problem Solving Model

## 1. Introduction of the problem:

Classical conversation with teacher to give meaning to the problem

## 2. Students working on the problem:

Students work individually or in pairs. (Teacher walks and looks around, helps and chooses problem solving approaches that are useful for the classroom discussion).

## 3. Classroom discussion:

Teacher and pupils understand, compare, check and evaluate their problem solving approaches and reflect on learning outcomes at the end.

(Stein et al., 2008)

The teacher has a hard job to realize this in a good way

# Mathematical problem solving in primary education



Larsson & Ryve (2011) wrote about this:

The teacher must be able to handle a wide spectrum of student solutions in a way that makes the whole class advance.

He must anticipate student responses, monitor student responses during the explore phase, select student responses for whole class discussion, sequence student responses and connect student responses to each other and to powerful mathematical ideas.

It is hard to effectively orchestrate mathematical whole class discussions.

# Mathematical problem solving in primary education



*The classroom discussion runs better and is more productive if the teacher has already explored what can happen.*

Prepare your problem-solving lessons thoroughly, by:

- (1) solving the problem in multiple ways,
- (2) predicting mistakes of students,
- (3) devising hints and support,
- (4) selecting and order problem-solving approaches to be used in the whole class discussion and
- (5) predicting learning outcomes.

(Stein et al., 2008)

# Mathematical problem solving in primary education



Grandma calls all her chickens and her cat. All 20 legs come running.  
How many chickens does she have?



A non-routine problem for students of 8 – 9 years.

Lesson preparation:

- (1) solving the problem in multiple ways,
- (2) predicting mistakes of students,
- (3) devising hints and support,
- (4) selecting and order problem-solving approaches to be used in the whole class discussion and
- (5) predicting learning outcomes.

(Stein et al., 2008)

# Mathematical problem solving in primary education



Grandma calls all her chickens and her cat. All 20 legs come running. How many chickens does she have?



Kippen = chickens  
 Kat = cat  
 Poten = legs

Handwritten student work on a grey background. At the top, there are 20 vertical lines representing legs, grouped into pairs with red arcs. The groups are numbered 1 through 8, and the last group is labeled 'Kat'. Below this is a table:

Kippen	Kat	poten
1	1	6
2	1	8
3	1	10
4	1	12
5	1	14
6	1	16
7	1	18
8	1	20

Next to the table are two subtraction problems:

$$10 \text{ ki} - 20$$

$$1 \text{ ka} - 4$$


---


$$24$$

$$9 \text{ ki} - 18$$

$$1 \text{ ka} - 4$$


---


$$22$$

A boxed area contains another subtraction problem:

$$8 \text{ ki} - 16$$

$$1 \text{ ka} - 4$$


---


$$20$$

At the bottom, there are calculations in green and blue ink:

$$20 - 4 = 16$$

$$16 : 2 = 8 \text{ kippen}$$

Below that, in blue ink: *10 kippen hebben 20 poten*

Can you understand the problem solving approaches of these students?

Which mistakes did you predict?

Which of these problem solving approaches would you select for the whole class discussion?

What can students learn from it?

How useful was your extensive lesson preparation?

This is a low floor high ceiling problem (Boaler, 2016)

# Mathematical problem solving in primary education



## Teaching problem solving requires:

- non-routine, low-floor-high-ceiling problems
- thorough lesson preparation
- lessons consisting of three phases
  1. Introduction of the problem
  2. Students working on the problem
  3. Classroom discussion
- **teacher skills**

# Mathematical problem solving in primary education

The teacher...

- encourages students to interpret the problem in their own words.
- offers students hints, support and advice without taking away the thinking.
- asks students to present their problem solving approaches, choose them deliberately and in a thoughtful order
- supports students in clearly and completely articulating and presenting their problem solving approaches
- challenges students to follow, complete and/or represent in their own words the problem solving approaches of their fellow students
- invites students to ask each other questions
- gives students sufficient time to think.
- checks if everyone has understand the discussed problem solving approach.
- asks students to investigate and explain whether a discussed problem solving approach is correctly chosen, as well as correctly executed
- encourages students to compare two problem solving approaches, to identify similarities and differences.
- invites students evaluate each other's problem solving approaches - respectfully-, name pros and cons, and explain their views.
- asks students to evaluate an incorrect or a new point of view. "I know a student who did this ... What do you think?"
- reflects with students on their problem solving process. What has worked for you?
- asks students to verbalize what they have learned.

Sources: Golden (2010); Hill (2014); NCTM (1991); Larsson & Ryve (2011)



# Mathematical problem solving in primary education



It takes 15 minutes to roast a 1-pound turkey. How long does it take to roast a 24-pound turkey?



# Mathematical problem solving in primary education

Which teacher skills of our list do you recognize in the behavior of teacher Dana?

Are there any skills to extend the list of teaching skills?

The teacher...

- encourages students to interpret the problem in their own words.
- offers students hints, support and advice without taking away the thinking.
- asks students to present their problem solving approaches, choose them deliberately and in a thoughtful order
- supports students in clearly and completely articulating and presenting their problem solving approaches
- challenges students to follow, complete, and/or represent in their own words the problem solving approaches of their fellow students
- invites students to ask each other questions
- gives students sufficient time to think.
- checks if everyone has understand the discussed problem solving approach.
- asks students to investigate and explain whether a discussed problem solving approach is correctly selected, as well as correctly executed
- encourages students to compare two solution approaches, to identify similarities and differences.
- invites students to evaluate each other's problem solving approaches - respectfully-, name pros and cons, and explain their views.
- asks students to evaluate an incorrect or a new point of view. "I know a student who did this ... What do you think?"
- reflects with students on their problem solving process. What has worked for you?
- asks students to verbalize what they have learned.

# Mathematical problem solving in primary education



How did all this come together in my course on mathematical problem solving in primary education?

I provided my pre-service teachers with theory about:

- the importance of working on problem solving in primary education
- features of suitable non-routine mathproblems (low-floor-high-ceiling problems)
- the three phases in a problem solving lesson according to Stein et al. (2008)
- the five steps of a thorough lesson preparation according to Stein et al. (2008)
- teachers skills for teaching problem solving

# Mathematical problem solving in primary education



How did all this come together in my course on mathematical problem solving in primary education?

Task to be performed in **practice**:

- Choose three non-routine problems from a given selection
- Prepare three lessons on these problems using the five steps of Stein et al. (2008)
- Choose three teacher skills from the list, as personal learning objectives
- Teach the three lessons, follow the three phases of Stein et al. (2008)
- Let your practice supervisor observe, evaluate and produce feedback concerning your three chosen learning objectives
- Reflect on your personal growth with respect to the three chosen teacher skills

# Mathematical problem solving in primary education



Quotations from the course evaluation

## What have you learned during the course?

- I find this much harder than teaching “ordinary” mathematics lessons.
- I thoughtfully prepared the lessons, which gave me support during teaching. I can now predict children’s problem solving approaches, and better respond to errors and offer help.
- I found it difficult not to take the thinking away from the students. Often I was already taking steps in a conversation that the student hadn't actually taken yet. This is what I need to work on in the future.
- I am used to asking questions to get answers and through this course I realize that I need to ask more process-oriented questions.
- At first I was not satisfied about what I learned in this course. During teaching, the verbalizing of the children didn't go so well and I started filling in too much. But I tried and practiced it again. During the third lesson, I was able to stimulate and guide children better. So in the end I am happy with my development, but I still have to learn much more about teaching problem solving.

# Mathematical problem solving in primary education



How helpful was the feedback from your practice supervisor?

My practice supervisor prefers to give general feedback, for example on classroom management. If she has to give feedback on such specific learning objectives she has to pay attention to too many small things.

I gave my practice trainer the document (on teacher skills), but I feel like she hasn't really studied it.

# Mathematical problem solving in primary education



Do non-routine math problems deserve a place in primary education? (Discuss this also with your supervisor)

My practice supervisor said:

Students can learn a lot from such problems and discussions, but it takes too much time.  
You should not fall behind with the text book.

I find this meaningful, and observing your lessons I am amazed at what my students are capable of.

Unfortunately, the program does not provide enough room for these activities.

Pre-service teachers said:

After teaching these lessons, I am aware of how important it is to offer non-routine math problems regularly.

Teaching problem solving is valuable for each student, also weak students.

I am looking forward to providing my class with more non-routine math problems.

You can always find time for this if you want, for example, as a puzzle on Friday afternoon.

# Mathematical problem solving in primary education



Do non-routine math problems deserve a place in primary education?



pre-service teacher    in-service teacher

Do you have experiences in your country?

Do you have advice for me (the Netherlands)?

Thank you for your attention. I hope I have given you food for thought.



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# Mathematical problem solving in primary education



Ik heb een schat aan data:

- Lesvoorbereidingen
- Verslagen
- Reflecties
- Feedback van practice supervisor
- Evaluatie
- Enkele interviews

Ik heb dit alles nog niet goed kunnen analyseren

Maar ik wil toch enkele uitspraken uit de evaluatie en de interviews met u delen

# Mathematical problem solving in primary education



Uitspraken uit de schriftelijke evaluatie en enkele interviews

## Wat heb je ervan geleerd?

Ik heb meer moeite met het geven van deze lessen dan met 'normale' rekenlessen.

Ik heb de rekentekenplannen doordacht ingevuld en dat gaf houvast tijdens de les. Ik kan nu oplossingsmanieren van kinderen voorspellen, en beter inspelen op fouten en hulp bieden.

Ik vond het lastig om het denkwerk niet zomaar weg te nemen. Vaak was ik al stappen aan het zetten die de leerling eigenlijk nog moest doen. Hier moet ik in het vervolg dus nog aan werken.

Ik stelde eerst antwoordgerichte vragen en deze cursus heeft me doen realiseren dat ik meer procesgerichte vragen moet stellen

Eerst was ik niet tevreden met mijn ontwikkeling. Ik kon niet alle vragen stellen die ik wilde stellen en bij het presenteren van de oplossingsmanieren ging het verwoorden bij de kinderen niet zo goed en ging ik te veel invullen. Maar ik heb veel nieuwe dingen gedaan en geoefend. Bij de derde les kon ik kinderen beter stimuleren en begeleiden. Dus ik ben blij met mijn ontwikkeling, maar moet nog wel verder groeien.

# Mathematical problem solving in primary education



Hoe was de feedback van de praktijkopleider?

Mijn praktijkopleider geeft liever algemene feedback, bijv. over klassenmanagement.

Als ze feedback moet geven op zulke specifieke leerdoelen moet ze op te veel dingen tegelijk letten.

Ik heb mijn praktijkopleider het document (over leerkrachtvaardigheden) gegeven, maar ik had niet het idee dat ze het echt had doorgenomen.

# Mathematical problem solving in primary education



Verdiene non-routine rekenopgaven een plaats in het basisonderwijs?

Praktijkopleiders:

Leerlingen kunnen veel leren van zo'n opdracht, maar het kost wel veel tijd. Je moet niet achterop raken met de methode.

Mijn praktijkopleider vond het betekenisvol, was verrast over wat kinderen konden.

Het programma biedt helaas niet genoeg ruimte voor deze activiteiten.

Studenten

Na het geven van de lessen ben ik me bewust hoe belangrijk het is non-routine rekenproblemen vaker aan te bieden

Het is voor iedereen goed, ook voor zwakke rekenaars

Ik heb zin om deze klas meer non-routine rekenproblemen aan te bieden

Je kunt hier altijd wel ruimte vinden. Bijvoorbeeld als puzzel op de vrijdagmiddag.

# Mathematical HOTS in primary education

Ways to enrich your mathematics textbook



Misschien  
weglaten?

## Small-group discussion

Do the primary mathematics textbooks in your country offer opportunities to learn mathematical higher order thinking skills?

If they do, can you describe examples of tasks or problems?

If they don't, do you know what primary teachers do to enrich their maths education?